- Assignment 7 due on February 25
- Assignment 8 due on March 4
- Test 4 opens on March 12

- Today: Sequences
- Next: Reading Week!
- After Reading Week: Properties of sequences (**Watch Videos 11.3, 11.4**)

Warm up

Write a formula for the general term of these sequences

1. $\{r_n\}_{n=0}^{\infty} = \{\, 1,\ 4,\ 9,\ 16,\ 25,\ \ldots \}$

2. $\{s_n\}_{n=1}^{\infty} = \{\, 1,\ -2,\ 4,\ -8,\ 16,\ -32,\ \ldots \}$

3. $\{m_n\}_{n=1}^{\infty} = \left\{ \frac{2}{1!}, \frac{3}{2!}, \frac{4}{3!}, \frac{5}{4!}, \ldots \right\}$

4. $\{j_n\}_{n=1}^{\infty} = \{\, 1,\ 4,\ 7,\ 10,\ 13,\ \ldots \}$

Sequences vs functions – convergence

For any function f with domain $[0, \infty)$,
we define a sequence as $a_n = f(n)$.
Let $L \in \mathbb{R}$. Which of these implications is true?

1. IF $\lim_{x \to \infty} f(x) = L$, THEN $\lim_{n \to \infty} a_n = L$.

2. IF $\lim_{n \to \infty} a_n = L$, THEN $\lim_{x \to \infty} f(x) = L$.

3. IF $\lim_{n \to \infty} a_n = L$, THEN $\lim_{n \to \infty} a_{n+1} = L$.

Definition of limit of a sequence

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence. Let $L \in \mathbb{R}$.
Which statements are equivalent to "$\{a_n\}_{n=0}^{\infty} \longrightarrow L$"?

1. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N},\ \forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < \varepsilon.$
2. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N},\ \forall n \in \mathbb{N},\quad n > n_0 \implies |L - a_n| < \varepsilon.$
3. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{R},\ \forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < \varepsilon.$
4. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N},\ \forall n \in \mathbb{R},\quad n \geq n_0 \implies |L - a_n| < \varepsilon.$
5. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N},\ \forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| \leq \varepsilon.$

6. $\forall \varepsilon \in (0,1),\ \exists n_0 \in \mathbb{N},\ \forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < \varepsilon.$
7. $\forall \varepsilon > 0,\quad \exists n_0 \in \mathbb{N},\ \forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < \frac{1}{\varepsilon}.$
8. $\forall k \in \mathbb{Z}^+,\quad \exists n_0 \in \mathbb{N},\ \forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < k.$
9. $\forall k \in \mathbb{Z}^+,\quad \exists n_0 \in \mathbb{N},\ \forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < \frac{1}{k}.$