MAT137 - Calculus with proofs

- **Deadline to add/change courses:** today
- TODAY: More proofs
- FRI: Abs values and distances (**Video 2.4**)
- MON: Limits (Videos 2.1, 2.2, 2.3)

Variations on induction

Let S_n be a statement depending on a positive integer n.

In each of the following cases, which statements are guaranteed to be true?

1. We have proven:
   - S_3
   - $\forall n \geq 1,\ S_n \implies S_{n+1}$
2. We have proven:
   - S_1
   - $\forall n \geq 3,\ S_n \implies S_{n+1}$
3. We have proven:
   - S_1
   - $\forall n \geq 1,\ S_n \implies S_{n+3}$
4. We have proven:
   - S_1
   - $\forall n \geq 1,\ S_{n+1} \implies S_n$

Variations on induction 2

We want to prove

$$\forall n \geq 1,\ S_n$$

So far we have proven

- S_1
- $\forall n \geq 1,\ S_n \implies S_{n+3}.$

What else do we need to do?

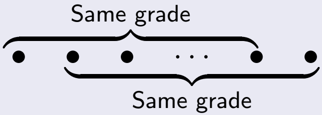
What is wrong with this proof by induction?

Theorem

$\forall N \geq 1$, *every set of* N *students in MAT137 will get the same grade.*

Proof.

- **Base case.** It is clearly true for $N = 1$.
- **Induction step.**
  Assume it is true for N. I'll show it is true for $N + 1$.
  Take a set of $N + 1$ students. By induction hypothesis:
  - The first N students get the same grade.
  - The last N students get the same grade.



Hence the $N + 1$ students all get the same grade.

□

What is wrong with this proof by induction?

For every $N \geq 1$, let

$$S_N = \text{"every set of } N \text{ students in MAT137 will get the same grade"}$$

What did we actually prove in the previous page?

- S_1 ?
- $\forall N \geq 1, \quad S_N \implies S_{N+1}$?

What is wrong with this proof? (1)

Theorem

The sum of two odd numbers is even.

Proof.

3 is odd.

5 is odd.

$3 + 5 = 8$ is even. □

What is wrong with this proof? (2)

Theorem

The sum of two odd numbers is always even.

Proof.

$x = 2a + 1$ odd

$y = 2b + 1$ odd

$x + y = 2n$ even

$2a + 1 + 2b + 1 = 2n$

$2a + 2b + 2 = 2n$

$a + b + 1 = n$ $\square$