MAT 137Y: Calculus with proofs
Assignment 7 - Comments and common errors

Global comments

Please review Video 8.7 to understand the difference between "the three types of integrals".

Q1a

- Please distinguish between indefinite integrals

$$\int t^n e^t dt \;=\; t^n e^t \;-\; n \int t^{n-1} e^t dt$$

  and definite integrals

$$\int_0^x t^n e^t dt \;=\; t^n e^t \Big|_{t=0}^{t=x} \;-\; n \int_0^x t^{n-1} e^t dt$$

Q1b

- We asked you to prove an identity for all natural numbers n. If you want to start your induction at $n = 1$, that is okay, but then you still to write a separate proof for the case $n = 0$.
- In the induction step, you fix an arbitrary value of n and you assume the result to be true for this single value of n (and go on to prove it for $n + 1$). **If, instead, you assume the result to be true for all n, then you are assuming the conclusion of the proof. That is completely wrong** (and you get 0 points).

Q1c

- $-1^n \neq (-1)^n$. Rather $-1^n = -(1^n) = -1$.
- Noticing a pattern is not a proof. Your pattern may still be wrong (or only work for the first few values of n). You need to prove it.
- A recursive formula is writing λ_n in terms of λ_{n-1}. An explicit formula is writing λ_n in terms of n and *no other λ's*.

Q2

- **When you use substitution in a definite integral, you need to change the bounds of integration.**
- If $G(x) = \displaystyle\int_a^x g(t)dt$, then you can immediately write $\displaystyle\int g(x)dx = G(x) + C$

  If you do not understand why, or you think this needs justification, rewatch Video 8.7 again.

- Rewatch the role of dummy variables on Video 8.7:

$$\int_a^b f(x)dx = \int_a^b f(u)du \qquad \text{but} \qquad \int f(x)dx \neq \int f(u)du.$$

- Your answers to Q2a and Q2c should be functions of x. Your answers to Q2b and Q2d should be functions of x plus arbitrary constants. if you answer, for example "$F_p - F_{p+2}$" or "$(-1)^{p+1}F_p$" or even "$(-1)^{p+1}F_p(\ \ )$", then your answer is not only wrong: it is nonsense.

Q3

- When you have an integral inside another integral, the order of the two differentials matter. For example, it makes sense to write

$$\int_a^b \int_x^b \mu(x)\mu(y)(x-y)^2 dydx$$

It actually means

$$\int_a^b \int_x^b \mu(x)\mu(y)(x-y)^2 dydx = \int_a^b \left(\int_x^b \mu(x)\mu(y)(x-y)^2 dy\right) dx = \int_a^b f(x)dx$$

for the function f defined as

$$f(x) = \int_x^b \mu(x)\mu(y)(x-y)^2 dy$$

This is a valid way to define the function f, so it is all good.

By contrast, we cannot write

$$\int_a^b \int_x^b \mu(x)\mu(y)(x-y)^2 dxdy$$

Try to make sense of it. It should mean something like this

$$\int_a^b \int_x^b \mu(x)\mu(y)(x-y)^2 dxdy = \int_a^b \left(\int_x^b \mu(x)\mu(y)(x-y)^2 dx\right) dy = \int_a^b g(y)dy$$

for the "function" g defined as

$$g(y) = \int_x^b \mu(x)\mu(y)(x-y)^2 dx$$

But this is not a valid way to define a function