MAT 137Y: Calculus with proofs
Assignment 5 - Comments and common errors

Q1

- Finding the critical points of a function is NEVER enough to find the global maximum or minimum. You need to justify why the function has a (global) maximum or minimum at that point.

Q2a

- Finding the values of A, B, and C that work is not enough. You also need to prove that no other values work.
- You may only use L'Hôpital's Rule when the limit of the quotient of derivatives exists, or is ∞, or is $-\infty$. Therefore, if in your argument at some point conclude that the limit of the quotient of derivatives simply "DNE", your use of L'Hôpital's Rule was illegal.

Q2b

- We asked for a proof by induction. That was to help you! If you attempt to do it "directly" by saying something like "use L'Hôpital's Rule n times" then you are hiding the induction step and your proof is not rigorous. In general, every proof where you write "do something n times" is probably a proof by induction in disguise, and the only way to make it rigorous is by using induction explicitly.
- The claim you need to prove by induction is something like

  $$S_n \ = \ \text{“for every function } f\text{, the limit } \ldots \text{ is 0”}$$

  If you did not include the "for every", then your "proof" is likely entirely wrong. If you fix one single function f, it is impossible to relate the "$n+1$-st limit for f" and the "n-th limit for f". Rather, it is only possible to relate one limit for f with the other limit for f'. That is why you need to include "for every function f" in the claim you are proving.
- Guessing the values for the coefficients by noticing the pattern in Q2a is not a proof.

Notational errors:

- The n-th derivative of f is $f^{(n)}$, not f^n.
- If P is the name of a polynomial, then $P(x)$ is its value at x. You can write $P(x) = 1 + x$, for example, but not $P = 1 + x$.
- The derivative of f at 0 is $f'(0)$, not $(f(0))'$. The latter is nonsense.

Q2c

- Review the definition of polynomial. If your answer involves e^x or $\sin x$ or $\cos x$, then it is not a polynomial.

Q3

- P, Q, and R are points in the graph of f. They are not values in the domain of f.
- When you use MVT, specify which function you are using, specify which interval you are using, and verify the hypotheses of the theorem before using it.
- f is not twice differentiable. You also do not need f to be twice differentiable. Why did some of you assume it?

Q4a

- We expected this question to be easy, but it was the worst question on the assignment. Many students just pushed symbols around making up all kinds of illegal operations without any respect for the limit laws. It looked a lot like bluffing.

  If you do not know an answer, do not make stuff up just to fish for marks. You will not get your marks, you will make a TA very sad, and you will lose your dignity.

- As an example, the following answer is not only wrong: it is an abomination.

  "We know
  $$\lim_{x\to\infty} [f(x) - (mx + b)] = 0$$
  Therefore
  $$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} (mx + b)$$
  and therefore
  $$\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{mx + b}{x} = m\text{"}$$
  You cannot go from the firs equationt to the second equation: what if the limits on the second equation do not exist?

  You cannot go from the second equation to the third equation. The limit laws only apply when each individual limit exists, and in this case you know for a fact they do not.

- This is a separate comment. In general, noticing that
  $$\lim_{x\to\infty} [f(x) - (mx + b)] = 0$$
  is by itself not enough to conclude that
  $$\lim_{x\to\infty} \frac{f(x) - (mx + b)}{\text{something else}} = 0.$$

Q4b

- A linear function is its own asymptote. This is not up for debate. Review the definition of asymptote.
- To prove a function does not have an asymptote, you need to show that *for every* $m, b \in \mathbb{R}$, the line $y = mx + b$ is not an asymptote. And you need to do this from the definition.

- You may only use limit laws when the individual limits exist.
- Overall, use definitions! In this case, use the definition of asymptote. We gave it to you for a reason.

Q4c

- The question specifically includes the assumption that $\lim_{x \to \infty} f(x) = \infty$. If your counterexample did not satisfy this, then it was not a counterexample.