

GEOMETRIC INEQUALITIES

Questions

1. Let π_k denote the projection onto the k -th coordinate hyperplane $P_k = \{x_k = 0\}$ in \mathbb{R}^n . Does there exist a constant C_n , such that for every convex compact set $K \subset \mathbb{R}^n$ the following inequality holds:

$$\min_k Area(\pi_k(K)) \leq C_n Vol(K)?$$

2. Let $C = D_r^2 \times [0, 1]$ denote the standard solid cylinder in \mathbb{R}^3 of height h and radius r . Suppose $\Phi : C \rightarrow V \subset \mathbb{R}^3$ is a diffeomorphism, such that $dist(\Phi(D_r^2 \times \{0\}), \Phi(D_r^2 \times \{1\})) = h$ and for every embedded disc $S \subset C$ with $\partial S \subset \partial D_r^2 \times [0, 1]$ we have $Area(\Phi(S)) \geq \pi r^2$.

Is it true that $Vol(\Phi(C)) \geq \pi r^2 h$? Can one construct a sequence of examples with ratio $\frac{Vol(\Phi(C))}{\pi r^2 h} \rightarrow 0$?