# Proof By Contraposition

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The *contrapositive* of the statement "$A \to B$" (i.e., "$A$ implies $B$.") is the statement "$\sim B \to \sim A$" (i.e., "$B$ is not true implies that $A$ is not true."). These two statements are equivalent. Therefore, if you show that the contrapositive is true, you have also shown that the original statement is true.

For example, instead of proving "$x$ being an integer implies that $x$ is a real number", we can prove that If $x$ is not a real number, it could not have been an integer.

Some other examples of statements and their contraposition:

<u>Example 1:</u>

*Statement:* Any two points in $\mathbb{R}^4$ are collinear.
(In other words, being in $\mathbb{R}^4$ implies that the points are collinear.)
*Contraposition:* If two points are not collinear, they could not have been in $\mathbb{R}^4$.

<u>Example 2:</u>

*Statement:* Squares are closed figures.
(If a figure is a square, then it is a closed figure.)
*Contrapositive:* A figure that is not closed cannot be a square.
(If the figure is not closed then the figure is not a square.)

<u>Example 3:</u>

*Statement:* If the function $f$ is an odd polynomial it has at least one root.
*Contraposition:* A function with no roots is not an odd polynomial.