(1) Let A be an orthogonal 3×3 matrix with $\det A > 0$.

Prove that for any $u, v \in \mathbb{R}^3$ we have that $Au \times Av = A(u \times v)$.

Hint: Prove this formula for orthonormal vectors u, v first.

You are not allowed to use results from appendix A1 of the book without proof.

(2) Give a full proof of the following result from class.

Let $\gamma(t)$ be a regular curve in $\mathbb{R}^3$ whose curvature is positive for all t.

Let A be an orthogonal 3×3 matrix with $\det A > 0$ and let v_0 be a point in $\mathbb{R}^3$. Let $F\colon \mathbb{R}^3 \to \mathbb{R}^3$ be a rigid motion given by the formula $F(v) = Av + v_0$.

Let $\tilde{\gamma}(t) = F(\gamma(t))$.

(a) Prove that $k(\gamma(t)) = k(\tilde{\gamma}(t))$ and $\tau(\gamma(t)) = \tau(\tilde{\gamma}(t))$ fot any t.

(b) what happens with the above formula if the matrix A is orthogonal with negative determinant?

(3) Let $\gamma(t)$ be a regular curve in $\mathbb{R}^3$ whose curvature is positive for all t.

Prove that $\gamma(t)$ lies in a plane if and only if $\gamma'(t), \gamma''(t), \gamma'''(t)$ are linearly dependent for all t.

(4) Let $\gamma\colon [a,b] \to \mathbb{R}^2$ be a unit speed curve. Let $g(t) = \gamma'(t)$ (it's also a curve in $\mathbb{R}^2$) and let k_t be the signed curvature of γ.

Prove that $\int_a^b k_t = \int_g xdy - ydx$

(5) Let γ be a simple closed curve in $\mathbb{R}^2$. Suppose its curvature is $\geqslant 1$ everywhere. Prove that the area of the interior of γ is $\leqslant \pi$.

(6) Recall that in homework 1 we considered the $\gamma(t)$ obtained by tracing a point on a circle of radius 1 rolling along a circle of radius 3 on the inside (see the image here `http://www.math.toronto.edu/vtk/363Winter2014/curve.gif`).

Find the area inside γ.