(1) Prove the following property of the cross product used in class:

$$a \times (b \times c) = (a, c)b - (a, b)c$$

for any $a, b, c \in \mathbb{R}^3$

(2) Let $f\colon \mathbb{R} \to \mathbb{R}$ be a smooth function and let $\gamma(t) = (t, f(t))$ be a prarameterization of the graph of f.

Prove that $k(\gamma(t_0)) = 0$ if and only if $f''(t_0) = 0$ where t_0 is any point in $\mathbb{R}$.

(3) Let $\gamma(t)$ be a regular curve in $\mathbb{R}^n$ and let A be an orthogonal $n \times n$ matrix (i.e $A \cdot A^t = Id$).

Let $\tilde{\gamma}(t) = A(\gamma(t))$. Prove that $k(\gamma(t)) = k(\tilde{\gamma}(t))$ for all t.

(4) (a) Let $\gamma_1\colon (-t_0 - \epsilon, t_0 + \epsilon) \to \mathbb{R}^3$ be a unit speed curve in $\mathbb{R}^3$. Suppose $k(\gamma_1(t_0)) > 0$.

Prove that there exists a unique unit speed circle $\gamma_2(t)$ such that $\gamma_1(t_0) = \gamma_2(t_2)$ and $\lim_{t\to t_0} \frac{|\gamma_1(t) - \gamma_2(t)|}{(t-t_0)^2} = 0$.

Here by a circle we mean a usual circle in some plane in $\mathbb{R}^3$.

(b) Conclude that for this circle $k(\gamma_1(t_0) = k(\gamma_2(t_0))$

(c) Find $\gamma_2(t)$ for $\gamma_1(t) = (\cos t, \sin t, t)$ and $t_0 = \pi/4$.