MAT 267F **Practice Term Test**

(1) Solve the following IVP:

$$\begin{cases} y' = 2\cos^2 y \cdot \sin(2x) \\ y(0) = 0 \end{cases}$$

What is the interval of existence of the solution?

(2) Using the variation of parameter find the general solution of the following equation:

$$y'' - y' - 2y = te^t$$

(3) Mark true or false. If true give an explanation. If false, give a counterexample.

(a) For any real 2×2 matrix A and any nonzero solution of $y' = Ay$ we either have that $||y(t)|| \to 0$ or $||y(t)|| \to \infty$ as $t \to \infty$.

(b) Suppose $y(t)$ is a nonzero solution of $y' = Ay$ for some 2×2 matrix A such that $y(t) \underset{t\to\infty}{\longrightarrow} 0$. Then if we change the initial condition $y(0)$ slightly, the solution will still go to 0 as $t \to \infty$.

In other words, there exists $\epsilon > 0$ such that if $\tilde{y}' = A\tilde{y}$ and $||y(0) - \tilde{y}(0)|| < \epsilon$ then $\tilde{y}(t) \underset{t\to\infty}{\longrightarrow} 0$.

(4) Find the general solution of the following system

$$\begin{cases} y_1' = -y_1 + 5y_2 \\ y_2' = -2y_1 - 3y_2 \end{cases}$$

What type of phase portrait does this system have?

(5) Find a 2×2 matrix A such that $y_1 = \begin{pmatrix} e^t \\ -e^t \end{pmatrix}$ and $y_2 = \begin{pmatrix} e^{2t} \\ -2e^{2t} \end{pmatrix}$ satisfy the equation $y' = Ay$.

(6) Find e^A for the following matrix

$$A = \begin{pmatrix} 2 & 1 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{pmatrix}$$

(7) Let A be an $n \times n$ matrix. Prove that $\det(e^A) = e^{trA}$.
(If you wish, you can assume that all the eigenvalues of A are distinct.)

(8) Show that if $e^{tA}e^{tB} = e^{tB}e^{tA}$ for any real t then $AB = BA$.
Hint: Differentiate!!