

### John Nash Problem

Show that given any infinitely differentiable functions  $M(x, y, z), N(x, y, z), P(x, y, z)$  on  $\mathbb{R}^3 \setminus \{0\}$  satisfying

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}, \quad \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}, \quad \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}$$

there exists a differentiable function  $H(x, y, z)$  on  $\mathbb{R}^3 \setminus \{0\}$  such that

$$\frac{\partial H}{\partial x} = M, \quad \frac{\partial H}{\partial y} = N, \quad \frac{\partial H}{\partial z} = P.$$