(1) Let $x(t_1,t_2) = t_1 e^{t_2}, y(t_1,t_2) = t_1^2 + \sin(t_1 t_2)$. Let $f(x,y)$ be a differentiable function $f\colon R^2 \to R$. Let $g(t_1,t_2) = f(x(t_1,t_2), y(t_1,t_2))$. Express $\frac{\partial g}{\partial t_1}(1,0)$ and $\frac{\partial g}{\partial t_2}(1,0)$ in terms of partial derivatives of $f$.

(2) Show that the following functions are differentiable and find their differentials
 (a) $f(x,y,z) = x^{y^z}$ where $x > 0, y > 0, z \in \mathbb{R}$
 (b) $f(x,y) = \int_{x^2}^{x+y} g(t)dt$ where $g\colon \mathbb{R} \to \mathbb{R}$ is continuous.

(3) Let $f, g\colon \mathbb{R}^n \to \mathbb{R}$ be differentiable at $a \in \mathbb{R}^n$. Using the definition of differentiability prove that $f \cdot g\colon \mathbb{R}^n \to \mathbb{R}$ is differentiable at $a$ and $d(fg)_a = f(a)dg_a + g(a)df_a$.
 *Hint:* Write $f(a+h)g(a+h) - f(a)g(a)$ as $f(a+h)g(a+h) - f(a)g(a) = f(a+h)g(a+h) - f(a+h)g(a) + f(a+h)g(a) - f(a)g(a) = f(a+h)(g(a+h) - g(a)) + g(a)(f(a+h) - f(a))$.

**Extra Credit Problem (to be written up and submitted separately)**

Let $A$ be an $n \times n$ matrix. Define $e^a$ by the formula

$$e^A = \sum_{k=0}^{\infty} \frac{A^k}{k!}$$

Prove that

(1) $e^A$ is well defined for any $n \times n$ matrix $A$.

(2) The function $A \mapsto e^A$ is continuous everywhere on the vector space of all $n \times n$ matrices identified with $\mathbb{R}^{n^2}$.

(3) The function $A \mapsto e^A$ is differentiable everywhere on the vector space of all $n \times n$ matrices.