(1) Let S be a rectifiable subset of the xz plane in $\mathbb{R}^3$ such that $Cl(S) \subset \{x > 0\}$. Let V be a solid obtained by rotating S around z axis. Prove that V is rectifiable and $vol(V) = 2\pi \int_S x$.

Hint: Use cylindrical coordinates.

(2) Let $n > 1$. Give an example of an $n \times n$ matrix A which preserves volume but is not orthogonal.

(3) Let$A$ be an $n \times n$ matrix with $\det A = 0$ and $S \subset \mathbb{R}^n$ be a rectifiable subset.

Prove that $A(S)$ has volume 0.

(4) Let $v_1, \ldots, v_n$ be n vectors in $\mathbb{R}^n$. Let B be an $n \times n$ matrix with $B_{ij} = \langle v_i, v_j \rangle$.

Prove that $\det B \geq 0$ and $\mathrm{volP(v_1, \ldots, v_n) = \sqrt{\det B}}$.

(5) Let $f \colon \mathbb{R} \to \mathbb{R}$ be given by $f(x) = |x|$. Prove that the graph of f is not a C^1 manifold.