MAT 257Y **Practice Term Test 3**

(1) Let $v_1, \dots, v_k \in \mathbb{R}^n$ where $n \geq k$. Prove that $\mathrm{vol}_k P(v_1, \dots, v_k) = 0$ if and only if $v_1, \dots, v_k$ are linearly dependent.

(2) Let T be a k-tensor on R^n. Prove that T is C^∞ as a map $\mathbb{R}^{nk} \to \mathbb{R}$.

(3) Let M be a union of x and y axis in $\mathbb{R}^2$. Prove that M is not a C^1 manifold.

(4) Prove that $S^2_+ = \{(x, y, z) \in \mathbb{R}^3 |$ such that $x^2 + y^2 + z^2 = 1, z \geq 0\}$ is a manifold with boundary.

(5) Let $c\colon [0, 1] \to (\mathbb{R}^n)^n$ be continuous. Suppose that $c^1(t), \dots, c^n(t)$ is a basis of $\mathbb{R}^n$ for any t.

Prove that $(c^1(0), \dots, c^n(0))$ and $(c^1(1), \dots, c^n(1))$ have the same orientation.

(6) Let C be the triangle in $\mathbb{R}^2$ with vertices $(0, 0), (1, 2), (-1, 3)$

Compute $\int_C x + y$.

(7) Let e_1, e_2 be a basis of a vector space V of dimension 2. Let $T \in \mathcal{L}^2(V)$ be given by $e_1^* \otimes e_1^* + e_2^* \otimes e_2^*$.

Prove that T can not be written as $S \otimes U$ with $S, U \in \mathcal{L}^1(V)$.

(8) Let $U \subset R^n$ be open. Let $f, g\colon U \to R$ be continuous and $|f| \leq g$. Suppose $\int_U^{ext} g$ exists.

Prove that $\int_U^{ext} f$ also exists.