Practice test

(1) Give the definitions of the following notions.
  (a) an open set in R^n;
  (b) a boundary point of a set $A \subset R^n$;
  (c) a function $f\colon R^n \to R^m$ differentiable at a point p;
  (d) a directional derivative of a function $f\colon R^n \to R^m$ at a point p.

(2) Find the partial derivatives of the following functions
  (a)

$$f(x,y) = \int_x^{\int_x^y g(t)dt} g(t)dt$$

Hint: put $F(x,y) = \int_x^y g(t)dt$ and express f as a composition.

  (b) $f(x,y) = \ln(\sin(x+y^2)^{\cos 2x})$

(3) Let $f\colon R^2 \to R$ be given by the formula

$$f(x,y) = \begin{cases} \frac{x^2y}{x^2+y^2} \text{ if } (x,y) \neq (0,0) \\ 0 \text{ if } (x,y) = (0,0) \end{cases}$$

  (a) Show that $f(x,y)$ is continuous at $(0,0)$.
  (b) Show that f has partial derivatives at $(0,0)$.
  (c) Does f has directional derivatives at $(0,0)$ in all directions?
  (d) Show that f is not differentiable at $(0,0)$.

(4) Show that a compact subset of R^n is bounded.

(5) let $f(x,y) = x^2 + 5y^2 - 4xy - 2y$. Find all possible points of minimum of $f(x,y)$.

(6) Let $f\colon R^n \to R^m$ be continuous.
Are the following statements true or false? Prove if true and give counterexamples if false.
  (a) If $A \subset R^n$ is closed and bounded then $f(A)$ is closed and bounded.
  (b) If $A \subset R^n$ is closed then $f(A)$ is closed.
  (c) If $A \subset R^n$ is bounded then $f(A)$ is bounded.

(7) Let $GL(n,R)$ be the set of all $n \times n$ invertible matrices.
Show that $GL(n,R)$ is open in R^{n^2}.