(1) Let $x(t_1, t_2) = t_1 e^{t_2}, y(t_1, t_2) = t_1^2 + \sin(t_1 t_2)$. Let $f(x, y)$ be a differentiable function $f\colon R^2 \to R$. Let $g(t_1, t_2) = f(x(t_1, t_2), y(t_1, t_2))$. Express $\frac{\partial g}{\partial t_1}(1, 0)$ and $\frac{\partial g}{\partial t_2}(1, 0)$ in terms of partial derivatives of f.

(2) Let $M(n)$ be the space of all $n \times n$ matrices. It can be identified with $\mathbb{R}^{n^2}$.

Let $f, g\colon \mathbb{R} \to M(n)$ be differentiable at t_0. Prove that $h(t) = f(t) \cdot g(t)$ is differentiable at t_0 and $h'(t_0) = f(t_0) \cdot g'(t_0) + f'(t_0) \cdot g(t_0)$.

(3) Show that the following functions are differentiable and find their differentials
  (a) $f(x, y, z) = x^{y^z}$ where $x > 0, y > 0, z \in \mathbb{R}$
  (b) $f(x, y) = \int_{x^2}^{x+y} g(t)dt$ where $g\colon \mathbb{R} \to \mathbb{R}$ is continuous.

Extra Credit Problem (to be written up and submitted separately)

Consider the map $f\colon GL(n, \mathbb{R}) \to M(n) = \mathbb{R}^{n^2}$ given by $f(A) = A^{-1}$. Prove that $df_{Id}(X) = -X$ for any $X \in M(n)$ where Id is the identity matrix.