

- (1) Let  $U \subset \mathbb{R}^n, V \subset \mathbb{R}^m$  be open. Show that  $U \times V \subset \mathbb{R}^{n+m}$  is open.
- (2) Let  $A \subset \mathbb{R}^n, B \subset \mathbb{R}^m$  be closed. Show that  $A \times B \subset \mathbb{R}^{n+m}$  is closed.
- (3) Let  $X$  be a metric space. let  $A \subset X$  be a subset of  $X$ . Prove that  $\text{Lim}(A)$  is closed.
- (4) Let  $X$  be a metric space. let  $A \subset X$  be a subset of  $X$ .
  - (a) Show that if  $A \subset C \subset X$  and  $C$  is closed then  $\text{Cl}(A) \subset C$ .
  - (b) Show that  $\text{Cl}(A)$  is equal to the intersection of all closed subsets of  $X$  containing  $A$ .
- (5) Let  $f: \mathbb{R}^n \rightarrow \mathbb{R}^m$  be a continuous map. Is it true that image of every closed set under  $f$  is closed? prove or give a counterexample.
- (6) Using only the definition of continuity show that if  $f: X \rightarrow \mathbb{R}^n$  and  $g: X \rightarrow \mathbb{R}^m$  are continuous then  $(f, g): X \rightarrow \mathbb{R}^{n+m}$  is continuous.
- (7) Find  $\text{br}(A), \text{Lim}(A)$  and  $\text{Cl}(A)$  for the following sets.
  - (a)  $A = \{0 < x^2 + y^2 \leq 1\} \subset \mathbb{R}^2$ .
  - (b)  $A = (0, 1) \times \{0\} \subset \mathbb{R}^2$ .
  - (c)  $A = \{(x, y) \in \mathbb{R}^2 \mid \text{such that } x > 0, y < \sin(1/x)\} \subset \mathbb{R}^2$ .
- (8) Let  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$  where  $X, Y, Z$  are metric spaces. Suppose  $f$  is continuous at  $p$  and  $g$  is continuous at  $f(p)$ . Using only the definition prove that  $g \circ f$  is continuous at  $p$ .
- (9) Let  $f, g: X \rightarrow \mathbb{R}$  are continuous at  $p$ . Using only the definition prove that  $f \cdot g: X \rightarrow \mathbb{R}$  is continuous at  $p$ .

**Extra Credit Problem (to be written up and submitted separately)**

Give an example of a nonempty set  $A \subset \mathbb{R}$  such that  $A = \text{br}(A) = \text{Lim}(A) = \text{Cl}(A)$ .