(1) Let V be a n-dimensional vector space. Let $T \in \mathcal{L}^k(V), S \in \mathcal{L}^l(V)$.

Prove that $Alt(T \otimes S) = Alt(Alt(T) \otimes Alt(S))$.

(2) Let V be a n-dimensional vector space and let $\langle \cdot, \cdot \rangle$ be a scalar product on V and let μ be an orientation on V.

Prove that there exists a unique alternating n-tensor $\omega \in \mathcal{A}^n(V)$ such that $\omega(e_1, \dots, e_n) = 1$ for any positively oriented orthonormal basis $e_1, \dots, e_n$ of V.

(3) Let Let $M \subset \mathbb{R}^3$ be given by $\{x^2 + y^2 - 5z^2 = 0\} \cap \{2x - y + z = 1\}$. Prove that M is a manifold and find T_pM for $p = (1, 2, 1)$.