(1) let T be a k-tensor on V. Prove that T is alternating if and only if $T(v_1, \dots v_k) = 0$ for any $v_1, \dots v_k$ such that $v_i = v_j$ for some $i \neq j$.

(2) let $\sigma \in S_{k+l}$ be the following permutation

$$\begin{pmatrix} 1 & \dots & k & k+1 & \dots & k+l \\ l+1 & \dots & l+k & 1 & \dots & l \end{pmatrix}$$

Prove that $sign(\sigma) = (-1)^{kl}$.

(3) Let $e_1, \dots e_n$ be a basis of a vector space V.
Let $T = (e_1^* + e_3^*) \otimes e_1^*$ and $S = (e_1^* + e_2^*) \otimes e_2^*$.
Compute $T \otimes S$ and $Alt(T \otimes S)$.

(4) Let $e_1, \dots, e_n$ be a basis of a vector space V.
Let $\eta, \theta \in \mathcal{A}^k(V)$. prove that $\eta = \theta$ iff $\eta(e_{i_1}, \dots, e_{i_k}) = \theta(e_{i_1}, \dots, e_{i_k})$ for any $1 \leq i_1 < i_2 < \dots < i_k$.

(5) Let $I = (i_1 < i_2 < \dots < i_k)$ and $J = (j_1 < j_2 < \dots < j_k)$ where $1 \leq i_l, j_s \leq n$. Prove that

$$e_{i_1}^* \wedge e_{i_2}^* \wedge \dots \wedge e_{i_k}^*(e_J) = \delta_{IJ}$$

(6) Prove that $\det(AB) = \det(A) \cdot \det(B)$ for any $n \times n$ matrices A, B.
Hint : Fix B and consider $f(A) = \det(AB)$.
Use that $\dim \Lambda^n(R^n) = 1$.

(7) Let $x, y, z \in \mathbb{R}^5$. Let

$$F(x, y, z) = 2x_2y_2z_1 + x_1y_5z_4, G(x, y) = x_1y_3 + x_3y_1, h(w) = w_1 - 2w_3$$

(a) Write $Alt(F)$ and $Alt(G)$ in terms of elementary alternating tensors.
(b) Write $Alt(F) \wedge h$ in terms of elementary alternating tensors.