(1) Let S be a rectifiable subset of the xz plane in R^3 such that $Cl(S) \subset \{x > 0\}$. Let V be a solid obtained by rotating S around z axis. Prove that V is rectifiable and $vol(V) = 2\pi \int_S x$.

Hint: Use cylindrical coordinates.

(2) Let $n > 1$. Give an example of an $n \times n$ matrix A which preserves volume but is not orthogonal.

(3) Finish the prove of the theorem from class and show that if A is an $n \times n$ matrix with $\det A = 0$ and $S \subset \mathbb{R}^n$ is rectifiable then $A(S)$ has volume 0.

(4) Let $v_1, v_2, \ldots v_k \in \mathbb{R}^n$. Let $v'_k = v_k + \sum_{i=1}^{k-1} \lambda_i v_i$ for some $\lambda_i \in \mathbb{R}$.

Prove that $\mathrm{vol_k(P(v_1, \ldots, v_{k-1}, v_k)) = vol_k(P(v_1, \ldots, v_{k-1}, v'_k))}$.

Extra Credit Problem (to be written up and submitted separately)

Give an example of a C^1 diffeomorphism $f\colon U \to V$ between open sets in $\mathbb{R}^n$ such that U and V are bounded, $||df||$ is bounded and U is rectifiable but V is not.

Is it possible to also have $||df^{-1}||$ bounded?