(1) Let U, V be open in $\mathbb{R}^n$ and $f\colon \mathbb{R}^n \to R$ be a function such that $\int_U^{ext} f$ and $\int_V^{ext} f$ exist.

Prove that $\int_{U\cap V}^{ext} f$ and $\int_{U\cup V}^{ext} f$ exist and

$$\int_{U\cup V}^{ext} f = \int_U^{ext} f + \int_V^{ext} f - \int_{U\cap V}^{ext} f$$

(2) Let $U = \{(x, y, z) \in \mathbb{R}^3 |$ such that $z > 0, x^2 + y^2 + z^2 < 1\}$.

Find $\int_U^{ext} z$ using spherical change of variables.

(3) Let $U = \{(x, y) \in R^2 | x^2 + y^2 < 1\}$. Let $f(x, y) = e^{x^2+y^2}$. Find $\int_U^{ext} f$.

(4) Finish the proof of Theorem from class:

Let $f\colon U \to V$ be a diffeomorphism between open subsets of R^n. Let $C \subset U$ be a compact subset.

Prove that $f(bd(C)) = bd(f(C))$.

Extra Credit Problem (to be written up and submitted separately)

Give an example of a diffeomorphism between open sets in R^n which is not C^1.

Hint: Look at the map $f\colon \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} 3x + x^2 \sin(1/x) \text{ if } x \neq 0 \\ 0 \text{ if } x = 0 \end{cases}$$