(1) Let V be a finite dimensional vector space over $\mathbb{C}$ with an inner product $\langle \cdot, \cdot \rangle$. Let $\langle \cdot, \cdot \rangle_2$ be a different inner product on V.

   (a) Given $v \in V$ show that there is a unique $v' \in V$ such that $\langle v, u \rangle_2 = \langle v', u \rangle$ for any $u \in V$.
   Define $T\colon V \to V$ by the formula $T(v) = v'$

   (b) Show that T is linear.

   (c) Show that T is self-adjoint with respect to $\langle \cdot, \cdot \rangle$.

   (d) Show that all eigenvalues of T are positive real numbers.

(2) Let V be a finite dimensional vector space and let $P_1, P_2\colon V \to V$ be projections (not necessarily orthogonal) such that $P_1 P_2 = P_2 P_1$.
   Prove that $P = P_1 P_2$ is again a projection.