EXTRA CREDIT.

To be written up and submitted separately from regular homework:

(1) Let $M_{n\times m}(F)$ be the space of $n \times m$ matrices with coefficients in F where $F = \mathbb{R}$ or $\mathbb{C}$. A function $A\colon [0,1] \to M_{n\times m}(F)$ is called continuous if $A_{ij}(t)$ is a continuous function of t for all i and j.

Let $n \geq 1$ and $r < n$. Let $V(n,r) \subset M_{n\times n}(\mathbb{R})$ be the set of all $n \times n$ real matrices of rank r.

Prove that $V(n,r)$ is path connected, i.e. for any $A_0, A_1 \in V(n,r)$ there exists a continuous map $A\colon [0,1] \to V(n,r)$ such that $A(0) = A_0$ and $A(1) = A_1$.