(1) Give a proof by induction of the following statement used class:

Let $m > 1$ be a natural number. Then for any $n \geqslant 0$ there exists an integer r such that $0 \leqslant r < m$ and $n \equiv r \pmod{m}$.

(2) (a) Find $2^{3^{100}} (\mod 5)$

(b) Find the last digit of $2^{3^{100}}$.

Hint: use part a) but remember that 10 is not prime.

(3) Using the Fundamental Theorem of Arithmetic prove that if $gcd(a,b) = 1$ and $a|bc$ then $a|c$.

(4) Find $1 + 2 + 2^2 + 2^3 + \ldots + 2^{219} (\mod 13)$.

(5) Prove the following result used in class.

Let $a = p_1^{k_1} \cdot \ldots p_m^{l_m}$ where all p_i are prime and $p_i \neq p_j$ for $i \neq j$. Suppose $p_1^{t_1}|a$ where t_1 is a nonnegative integer.

Prove that $t_1 \leqslant k_1$.