

- (1) Prove by mathematical induction that  $n^3 + 5n$  is divisible by 6 for any natural  $n$ .
- (2) Find the remainder when  $7^{101}$  is divided by 101.
- (3) Find the integer  $a$ ,  $0 \leq a \leq 20$  such that  $13a \equiv 1 \pmod{20}$ .
- (4) Prove that if  $m \equiv 1 \pmod{\phi(n)}$  and  $(a, n) = 1$  then  $a^m \equiv a \pmod{n}$ , where  $\phi$  is Euler's function.
- (5) Suppose  $3^{3^{100}}$  is written in ordinary way. What are the last two digits?
- (6) Prove that  $\sqrt[3]{\frac{2}{7}}$  is irrational.