(1) Let $P(z)$ be a polynomial with complex coefficients such that $P(n) = 0$ for all integer n.

Prove that $P(z) = 0$ for all $z \in \mathbb{C}$.

(2) Express the following complex number as $a + bi$ for some real a, b

$$z = (\frac{\overline{(2+3i) \cdot |1-3i|}}{1+\sqrt{2}i})^2$$

(3) Construct an explicit 1-1 and onto map $f\colon [0,1] \to (0,1)$.

(4) For any set S define $P(S)$ to be the set of all subsets of S. For example, if $S = \{a, b\}$ then $P(S) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$.

Let A be a finite set. Show that $|P(A)| = 2^{|A|}$.

Hint: Let $A = \{x_1, \dots, x_n\}$. Represent a subset S of A by a sequence of 0s and 1s of length n such that the i-th element in the sequence is 1 if $x_i \in S$ and is 0 if $x_i \notin S$.

(5) Prove that $|\mathbb{N}^k| = |\mathbb{N}|$ for all natural k.