

- (1) Let a, b be odd integers.
Prove that $\sqrt{a^2 + b^2}$ is irrational.
Hint: Look at divisibility by the powers of 2.
- (2) Finish the proof from class that $\sqrt{2} + \sqrt{3} + \sqrt{5}$ is irrational.
- (3) Prove that for any real numbers $a < b$ there exists an irrational number c such that $a < c < b$.
Hint: Look at the numbers of the form $q\sqrt{2}$ where q is rational.
- (4) Show that the equation

$$3x^3 + 2x^2 - 5x - 2 = 0$$

has no rational solutions.

- (5) Let $x = q_1\sqrt{3} + q_2\sqrt{5}$ where q_1 and q_2 are rational. Prove that x is rational if and only if $q_1 = q_2 = 0$.