(1) Let p_1, p_2, p_3 be distinct prime numbers.

Using the method from class give a careful proof of the formula

$$\phi(p_1^{k_1} p_2^{k_2} p_3^{k_3}) = (p_1^{k_1} - p_1^{k_1-1})(p_2^{k_2} - p_2^{k_2-1})(p_3^{k_3} - p_3^{k_3-1})$$

(2) Let a, b, c be natural numbers. Let (a, b, c) be the largest natural number that divides a, b and c.

  (a) Prove that $gcd(a, b, c) = gcd(gcd(a, b), c)$.

  (b) Prove that the equation $ax + by + cz = gcd(a, b, c)$ has an integer solution.

(3) Find $6^{100} (\mod 14)$.

Note: Note that $gcd(6, 14) \neq 1$!

(4) Suppose a and m be natural numbers such that $gcd(a, m) \neq 1$. prove that $a^{\phi(m)} \neq 1 (\mod m)$.