(1) Which of the following is a number field?
   (a) the set of all nonnegative rational numbers;
   (b) the set of numbers of the form $a+b\sqrt{2}+c\sqrt{3}$ where $a, b, c \in \mathbb{Q}$;
   (c) the set of numbers of the form $a + b\sqrt{2} + c\sqrt[4]{2} + d\sqrt[4]{8}$ where $a, b, c, d \in \mathbb{Q}$.
   Hint: Look at the appropriate tower of fields $\mathbb{Q} = F_0 \subset F_1 = \mathbb{Q}(\sqrt{2}) \subset F_2$
   (d) The set of irrational numbers.

(2) Let F be the field consisting of real numbers of the form $p+q\sqrt{2+\sqrt{2}}$ where p, q are of the form $a + b\sqrt{2}$, with a, b rational. Represent

$$\frac{1+\sqrt{2+\sqrt{2}}}{2-3\sqrt{2+\sqrt{2}}}$$

in this form.

(3) (a) Prove that $\sqrt[3]{\pi}$ is not constructible.
   (b) Prove that π^3 is not constructible.

(4) Find a polynomial with integer coefficients which has $1 + \sqrt{2} + \sqrt{3}$ as a root.

(5) Find the cardinality of the set of all surds.

(6) Is there a line in the plane such that every point on it is constructible?

(7) Let t be a transcendental number. Prove that the set $\{(a + bt) : a, b \in \mathbb{Q}\}$ is not a number field.