

- (1) Prove by mathematical induction that $n^3 + 5n$ is divisible by 6 for any natural n .
- (2) Find the remainder when 7^{101} is divided by 101.
- (3) Find the integer a , $0 \leq a \leq 20$ such that $13a \equiv 1 \pmod{20}$.
- (4) Prove that if $m \equiv 1 \pmod{\phi(n)}$ and $(a, n) = 1$ then $a^m \equiv a \pmod{n}$, where ϕ is Euler's function.
- (5) Suppose $3^{3^{100}}$ is written in ordinary way. What are the last two digits?
- (6) Prove that $\sqrt[3]{\frac{2}{7}}$ is irrational.