(1) Let a, b be odd integers.

Prove that $\sqrt{a^2 + b^2}$ is irrational.

Hint: Look at divisibility by the powers of 2.

- (2) Prove that $\sqrt{2} + \sqrt[3]{2}$ is irrational.
- (3) Prove that for any real numbers a < b there exists an irrational number c such that a < c < b.

Hint: Look at the numbers of the form $q\sqrt{2}$ where q is rational.

(4) Show that the equation

$$3x^3 + 2x^2 - 5x - 2 = 0$$

has no rational solutions.

(5) Prove that for any complex numbers z_1, z_2 such that $z_2 \neq 0$ we have

$$|\frac{z_1}{z_2}| = \frac{|z_1|}{|z_2|}$$

(6) Prove that for any complex numbers z_1, z_2, z_3 we have

$$(z_1 z_2) z_3 = z_1 (z_2 z_3)$$

(7) Express the following complex number as a + bi for some real a, b

$$z = \left(\frac{\overline{(2+3i)\cdot|1-3i|}}{1+\sqrt{2}i}\right)^2$$

(8) Let $z = \frac{(1+3i)^{150}}{(2+2i)^{50}(3+4i)^{75}}$ Find |z|.