

- (1) Let  $a, b, c$  be natural numbers. Let  $(a, b, c)$  be the largest natural number that divides  $a, b$  and  $c$ .
- (a) Prove that  $\gcd(a, b, c) = \gcd(\gcd(a, b), c)$ .
  - (b) Prove that the equation  $ax + by + cz = \gcd(a, b, c)$  has an integer solution.
- (2) Find  $6^{100} \pmod{14}$ .
- Note:* Note that  $\gcd(6, 14) \neq 1$ !
- (3) Prove that if  $m \equiv 1 \pmod{\phi(n)}$  and  $(a, n) = 1$  then  $a^m \equiv a \pmod{n}$ , where  $\phi$  is Euler's function.