(1) Give a proof by induction of the following theorem from class:

Let $m > 1$ be a natural number. Then for any $n \geqslant 0$ there exists an integer r such that $0 \leqslant r < m$ and $n \equiv r \pmod{m}$.

(2) Let p_1, p_2 be distinct primes. Using the Fundamental Theorem of Arithmetic prove that a natural number n is divisible by $p_1 p_2$ if and only if n is divisible by p_1 and n is divisible by p_2.

(3) Prime "triplets" are triples of prime numbers of the form $n, n+2, n+4$.

Find all prime triplets.

Hint: Think (mod 3).

(4) (a) Find all possible values of $2^k \pmod{6}$.

(b) Find all possible values of $k^2 \pmod{6}$

(5) Prove that for any natural k

$$4^k + 4 \cdot 9^k \equiv 0(\mod 5)$$

(6) Find the rule for checking when an integer is divisible by 13 similar to the rule for checking divisibility by 7 done in class.

(7) Prove that if $m > 1$ is not prime then there exist integers a, b, c such that $c \not\equiv 0(\mod m)$, $ac \equiv bc(\mod m)$ but $a \not\equiv b(\mod m)$.