(1) Finish the proof of the Shroeder-Berenstein Theorem from class.

Recall that the proof goes as follows. Let $f\colon S \to T$ and $g\colon T \to S$ be 1-1. Define S_S to be the set of points in S whose last ancestor is in S. Define S_T to be the set of points whose last ancestor is in T. Define S_∞ to be the set of points in S which have infinitely many ancestors. Define T_S, T_T, T_∞ similarly.

Then define $h\colon S \to T$ by the formula

$$h(s) = \begin{cases} f(x) \text{ if } s \in S_S \cup S_\infty \\ g^{-1}(s) \text{ if } s \in S_T \end{cases}$$

It was shown in class that $h\colon S_S \to T_S$ and $h\colon S_\infty \to T_\infty$ is 1-1 and onto.

Finish the proof by showing that $h\colon S_T \to T_T$ and $h\colon S_\infty \to T_\infty$ are also 1-1 and onto.

(2) Let $S = (0,1)$ and $T = [0,1)$. Let $f\colon S \to T$ be given by $f(x) = x$ and $g\colon T \to S$ be given by $g(x) = \frac{x+1}{2}$.
  (a) Find $S_S, S_T, S_\infty, T_S, T_T, T_\infty$
  (b) give an explicit formula for a 1-1 and onto map $h\colon S \to T$ coming from f and g using the proof of the Schroeder-Berenstein theorem.

(3) Let $T = \{1, 2, 3\}$.

Let S be the set of all functions $f\colon \mathbb{N} \to T$.

Prove that $|S| = |\mathbb{R}|$.

Hint: Use problem 7 from homework 8.

(4) Let S be an infinite set such that $|S| > |\mathbb{N}|$. Let $T \subset S$ be countable.
  (a) Prove that $S \backslash T$ is infinite.
  (b) Prove that $|S| = |S \backslash T|$.
  Hint: Construct $T' \subset S \backslash T$ such that T' is countable and use that $|T \cup T'| = |T|$ to construct an 1-1 and onto map from S to $S \backslash T$.
  (c) Find the cardinality of the set of transcendental numbers.

(5) Let S be the set of sequences $q_1, q_2, q_3, \ldots$ where q_i is real for every i and such that for every sequence there exists $n \in \mathbb{N}$ such that $q_i = 0$ for all $i \geq n$.

Prove that $|S| = |\mathbb{R}|$.

(6) Prove that $\sqrt{3} + \sqrt[3]{2}$ is algebraic.