(1) Solve the following quadratic equation
$z^2 + (1+i)z + i = 0.$

(2) Let z_0 be a root of $x^n = z$. Show that all roots of $x^n - z = 0$ have the form $z_0 \cdot \zeta_k$ where $\zeta_0, \dots \zeta_{n-1}$ are n-the roots of 1.

(3) Let $P(z)$ be a polynomial with real coefficients.
Prove that if z_0 is a root of $P(z)$ then $\bar{z_0}$ is also a root of $P(z)$.

(4) Let $P(z), Q(z)$ be two polynomials with complex coefficients such that $P(n) = Q(n)$ for any natural n.
Prove that $P(z) = Q(z)$ for all z.

(5) Express the following number as $a + bi$ for some real a, b:

$$\frac{(3-\sqrt{3}i)^{71}}{(1-i)^{53}}$$

(6) Find all complex solutions of the following equation

$$x^6 + 7x^3 - 8 = 0$$

(7) Express $\sin(3\theta)$ and $\sin(5\theta)$ as polynomials in $\sin\theta$.
Hint: Use De Moivre's formula and the identity $\sin^2\theta + \cos^2\theta = 1$.