

- (1) Prove that $\sqrt{2} + \sqrt[3]{2}$ is irrational.
- (2) Prove that for any real numbers $a < b$ there exists an irrational number c such that $a < c < b$.

Hint: Look at the numbers of the form $\frac{p}{q}\sqrt{2}$.

- (3) Show that the equation

$$3x^3 + 2x^2 - 5x - 2 = 0$$

has no rational solutions.

- (4) Let $x = q_1\sqrt{2} + q_2\sqrt{3}$ where q_1, q_2 are rational. Prove that x is rational if and only if $q_1 = q_2 = 0$.
- (5) Prove that for any complex numbers z_1, z_2, z_3 we have

$$(z_1 z_2) z_3 = z_1 (z_2 z_3)$$

- (6) Express the following complex number as $a + bi$ for some real a, b

$$z = \left(\frac{(2 + 3i) \cdot |1 - 3i|}{1 + \sqrt{2}i} \right)^2$$

- (7) Let $z = \frac{(1+3i)^{150}}{(2+2i)^{50}(3+4i)^{75}}$
Find $|z|$.