(1) Prove that if $m > 4$ is not prime then $(m-1)! \equiv 0(\mod m)$
(2) Find all possible values of $(n, n+6)$ where n is a natural number.
(3) (a) Let $(a,m) = 1$. Prove that there exists a natural number $b \leqslant m$ such that $ab \equiv 1(\mod m)$.
(b) Let $m = 15$. For every natural $a \leqslant 15$ satisfying $(a,m) = 1$ find a $b \leqslant 15$ such that $ab \equiv 1(\mod 15)$.
(4) Find the Euler function ϕ of each of the following numbers 48, 51, 101.
(5) Let p, q be distinct primes. Without using the general formula prove that $\phi(p^k q^l) = (p^k - p^{k-1})(q^l - q^{l-1})$.
(6) Find $10^{5^{101}}(\mod 21)$.
(7) Find $6^{100}(\mod 14)$.
Hint: Note that $(6,14) \neq 1$. Find $6^{100}(\mod 7)$ first. Then find an **even** number $a < 14$ such that $6^{100} \equiv a(\mod 7)$.
(8) Find $35^{25}(\mod 42)$.