(1) Give a proof by induction of the following theorem from class:

Let $m > 1$ be a natural number. Then for any $n \geqslant 0$ there exists an integer r such that $0 \leqslant r < m$ and $n \equiv r \mod m$.

(2) Prime "triplets" are triples of prime numbers of the form $n, n+2, n+4$.

Find all prime triplets.

Hint: Think (mod 3).

(3) (a) Show that there is no natural number k such that $2^k \equiv 1 (mod\ 6)$.
(b) Find all possible values of $2^k (mod\ 6)$.

(4) Prove that for any natural k

$$4^k + 4 \cdot 9^k \equiv 0(\mod 5)$$

(5) Find the rule for checking when an integer is divisible by 13 similar to the rule for checking divisibility by 7 done in class.

(6) Prove that if $m > 1$ is not prime then there exist integers a, b, c such that $c \not\equiv 0(\mod m)$, $ac \equiv bc(\mod m)$ but $a \not\equiv b(\mod m)$.

(7) Prove that if $m > 1$ is not prime then there exists $a \not\equiv 0(\mod m)$ such that $a^k \not\equiv 1(\mod m)$ of any natural k.

(8) Prove that for any natural n the number $11^{n+1} + 12^{2n-1} \equiv 0(\mod 133)$.