(1) Which of the following is a number field?
   (a) the set of all nonnegative rational numbers;
   (b) the set of numbers of the form $a + b\sqrt{2} + c\sqrt{3}$ where $a, b, c \in \mathbb{Q}$;
   (c) the set of numbers of the form $a + b\sqrt{2} + c\sqrt[4]{2} + d\sqrt[4]{8}$ where $a, b, c, d \in \mathbb{Q}$.
   Hint: Look at the appropriate tower of fields $\mathbb{Q} = F_0 \subset F_1 = \mathbb{Q}(\sqrt{2}) \subset F_2$
   (d) The set of irrational numbers.

(2) Let x_0 be a root of the polynomial $a_n x^n + \ldots a_1 x + 0$ where each a_i has the form $a_i = b_i + c_i\sqrt{2}$ where $b_i, c_i \in \mathbb{Q}$.
   Prove that x_0 is a root of a polynomial with rational coefficients.
   Hint: Write $f(x_0) = 0$, move all the terms with $\sqrt{2}$ to the right and square the sides.

(3) Let F be the field consisting of real numbers of the form $p+q\sqrt{2+\sqrt{2}}$ where p, q are of the form $a + b\sqrt{2}$, with a, b rational. Represent
$$\frac{1+\sqrt{2+\sqrt{2}}}{2-3\sqrt{2+\sqrt{2}}}$$
in this form.

(4) Find a tower of fields $\mathbb{Q} = F_0 \subset F_1 \subset F_2 \subset F_3$ such $F_1 = \mathbb{Q}(\sqrt{2})$, $F_2 = F_1(\sqrt{3})$ and $F_3 = F_2(\sqrt{1+\sqrt{2}+\sqrt{3}})$
   Show that all the steps in the tower except for the last one are nontrivial. I.e show that $F_0 \neq F_1$, and $F_1 \neq F_2$.