MAT 246S   **Practice Term Test**

(1) Prove by mathematical induction that $n^3 + 5n$ is divisible by 6 for any natural n.

(2) Find the remainder when 7^{101} is divided by 101.

(3) Find the integer a, $0 \le a \le 20$ such that $13a \equiv 1 (mod\ 20)$.

(4) Prove that if $m \equiv 1 (mod\ \phi(n))$ and $(a, n) = 1$ then $a^m \equiv a (mod\ n)$, where ϕ is Euler's function.

(5) Suppose $3^{3^{100}}$ is written in ordinary way. What are the last two digits?
    Hint: Use the previous problem.

(6) Prove that $\sqrt[3]{\frac{2}{7}}$ is irrational.

(7) Prove that

$$x = \sum_{n=1}^{\infty} \frac{1}{10^{n^2}}$$

is irrational.