(1) Prove that the set of finite subsets of $\mathbb{N}$ is countable.

(2) Let S be an infinite set such that $|S| > |\mathbb{N}|$. Let $T \subset S$ be countable.

  (a) Prove that $S \backslash T$ is infinite.

  (b) Prove that $|S| = |S \backslash T|$.
  Hint: Construct $T' \subset S \backslash T$ such that T' is countable and use that $|T \cup T'| = |T|$ to construct an 1-1 and onto map from S to $S \backslash T$.

  (c) Find the cardinality of the set of transcendental numbers.

(3) Let S be the set of sequences $q_1, q_2, q_3, \ldots$ where q_i is real for every i and such that for every sequence there exists $n \in \mathbb{N}$ such that $q_i = 0$ for all $i \geq n$.
Find the cardinality of S.

(4) Explain how to construct $\frac{2+\sqrt{3}}{3}$ using ruler and compass.

(5) Let $P(x)$ be a cubic polynomial with rational coefficients. Suppose it has a complex root of the form $a + bi$ where both a and b are rational.
Prove that $P(x)$ has a rational root.
Hint: Observe that $Q(x) = (x - a - bi)(x - a + bi)$ has rational coefficients. Divide $P(x)$ by $Q(x)$ and show that the remainder must be zero.

(6) Show that if $\sin\alpha$ and $\sin\beta$ are constructible then $\sin(\alpha + \beta)$ is also constructible.

(7) Let x_0 be a root of the polynomial $a_n x^n + \ldots a_1 x + 0$ where each a_i has the form $a_i = b_i + c_i\sqrt{2}$ where $b_i, c_i \in \mathbb{Q}$.
Prove that x_0 is a root of a polynomial with rational coefficients.
Hint: Write $f(x_0) = 0$, move all the terms with $\sqrt{2}$ to the right and square the sides.