(1) Finish the proof of Shroeder-Berenstein Theorem from class.

Recall that the proof goes as follows. Let $f: S \to T$ and $g: T \to S$ be 1-1. Define S_S to be the set of points in S whose last ancestor is in S. Define S_T to be the set of points whose last ancestor is in T. Define S_{∞} to be the set of points in S which have infinitely many ancestors. Define T_S, T_T, T_{∞} similarly.

Then define $h: S \to T$ by the formula

$$h(s) = \begin{cases} f(x) \text{ if } s \in S_S \cup S_\infty \\ g^{-1}(s) \text{ if } s \in S_T \end{cases}$$

It was shown in class that $h: S_S \to T_S$ and $h: S_\infty \to T_\infty$ are 1-1 and onto.

Finish the proof by showing that $h: S_T \to T_T$ is also 1-1 and onto. (2) Let S be infinite and $A \subset S$ be finite.

- Prove that $|S| = |S \setminus A|$.
- (3) Let S = (0, 1) and T = [0, 1). Let $f: S \to T$ be given by f(x) = x and $g: T \to S$ be given by $g(x) = \frac{x+1}{2}$.
 - (a) Find $S_S, S_T, S_\infty, T_S, T_T, T_\infty$
 - (b) give an explicit formula for a 1-1 and onto map $h: S \to T$ coming from f and g using the proof of the Schroeder-Berenstein theorem.
- (4) Prove that the set of functions $f \colon \mathbb{R} \to \mathbb{R}$ has cardinality bigger than \mathbb{R} .
- (5) Let $S = P(\mathbb{N})$

Show that $|S| = |\mathbb{R}|$.

Hint: It was shown on the last homework that $|S| \leq |\mathbb{R}|$. By Shroeder-Berenstein it's enough to show that $|\mathbb{R}| \leq |S|$. To do this we need to construct a 1-1 map $f: \mathbb{R} \to S$. Define f(x) by using the decimal expansion of x.

(6) Prove that $|\mathbb{R} \times \mathbb{N}| = |\mathbb{R}|$.