(1) Solve the following quadratic equation

$z^2 + (1 + i)z + i = 0.$

(2) Let z_0 be a root of $x^n = z$. Show that all roots of $x^n - z = 0$ have the form $z_0 \cdot \zeta_k$ where $\zeta_0, \dots \zeta_{n-1}$ are n-the roots of 1.

(3) Prove that $|\mathbb{N}^k| = |\mathbb{N}$ for any natural k.

Hint: Use induction.

(4) Let $\mathbb{Z}$ be the set of all integers.

Prove that $|\mathbb{Z}| = |\mathbb{N}|$.

(5) For any set S define $P(S)$ to be the set of all subsets of S. for example, if $S = \{a, b\}$ then $P(S) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$.

Let A be a finite set. Show that $|P(A)| = 2^{|A|}$.

Hint: Let $A = \{x_1, \dots, x_n\}$. Represent a subset S of A by a sequence of 0s and 1s of length n such that the i-th element in the sequence is 1 if $x_i \in S$ and is 0 if $x_i \notin S$.

(6) Let S be an infinite set. Prove that $|S| \geq |\mathbb{N}|$.

(7) Let $S = P(\mathbb{N})$

Show that $|S| \leq |\mathbb{R}|$.

Hint: represent a subset A of N as a sequence of 1s and 0s such that the n-th element of the sequence is 1 if $n \in A$ and is 0 if $n \notin A$.