(1) Find all prime numbers smaller than 100.

(2) Give a proof by induction (instead of a proof by contradiction given in class) that any natural number > 1 has a unique (up to order) factorization as a product of primes.

(3) Give a proof by induction that if $a \equiv b(\mod m)$ then $a^n \equiv b^n(\mod m)$ for any $n \geq 1$.

(4) Give a proof by induction of the following theorem from class:

Let $m > 1$ be a natural number. Then for any $n \geq 0$ there exists a natural number r such that $0 \leq r < m$ and $n \equiv r \mod m$.

(5) Prime "triplets" are triples of prime numbers of the form $n, n+2, n+4$.

Find all prime triplets.

Hint: Think (mod 3).

(6) Show that there is no natural number k such that $2^k \equiv 1(mod\ 6)$. Find all possible values of $2^k(mod\ 6)$.

(7) Prove that for any natural k

$$4^k + 4 \cdot 9^k \equiv 0(\mod 5)$$

(8) Find the rule for checking when an integer is divisible by 7 similar to the rule for checking divisibility by 11 done in class.