- (1) Prove that  $\frac{\sqrt{2}+\sqrt[3]{5}}{6}$  is not constructible. (2) Prove that  $\frac{\pi^2}{3}$  is not constructible.
- (3) Let F be the field consisting of real numbers of the form  $p+q\sqrt{2+\sqrt{2}}$ where p, q are of the form  $a + b\sqrt{2}$ , with a, b rational.. Represent

$$\frac{1+\sqrt{2}+\sqrt{2}}{2-3\sqrt{2}+\sqrt{2}}$$

in this form.

(4) Find a tower of fields  $Q = F_0 \subset F_1 \subset F_2 \subset F_3$  such that

$$\sqrt{1+\sqrt{2}+\sqrt{3}} \in F_3$$

Show that all the steps in the tower except for the last one are nontrivial. I.e show that  $F_0 \neq F_1$ , and  $F_1 \neq F_2$ . (5) Show that none of the real roots of  $3x^3 - 2x^2 - 2 = 0$  are constructible.

- (6) Let  $0 < \theta < \pi/2$  be the angle with  $\cos \theta = \frac{2}{7}$ . Show that  $\theta$  is constructible but  $\theta/3$  is not.
- (7) Show that the equation

 $x^9 - 4x^3 + 1 = 0$  has no constructible roots.