(1) Prove that

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\dots\frac{1}{n(n+1)}=\frac{n}{n+1}$$

(2) Prove that

$$\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\ldots+\frac{n}{2^n}=2-\frac{n+2}{2^n}$$

(3) Prove that

$$1+2q+3q^2+\ldots+nq^{n-1}=\frac{1-(n+1)q^n+nq^{n+1}}{(1-q)^2}$$

(4) Find the sum of the following geometric progression

$$\frac{x}{1+x^2}+\frac{x^2}{(1+x^2)^2}+\ldots+\frac{x^n}{(1+x^2)^n}$$

(5) Prove that

$$1^2+3^2+\ldots+(2n+1)^2=\frac{(n+1)(2n+1)(2n+3)}{3}$$

(6) Let $x_1>2$. Define x_n by the formula $x_{n+1}=\frac{3x_n+2}{x_n+2}$
Prove that $x_n>2$ for all n.

(7) Prove that $(1+p)^n>1+np$ for any real $p\neq 0, p>-1$ and any natural $n\geq 2$.

(8) Find and prove the formula for the sum

$$1^3+2^3+\ldots+n^3$$

(9) Find a mistake in the following "proof".

Claim. any two natural numbers are equal.

We'll prove the following statement by induction in n: Any two natural numbers $\leq n$ are equal.

We prove it by induction in n.

a) The statement is trivially true for $n=1$.

b) Suppose it's true for $n\geq 1$. Let a,b be two natural numbers $\leq n+1$. Then $a-1\leq n$ and $b-1\leq n$. Therefore, by induction assumption

$$a-1=b-1$$

Adding 1 to both sides of the above equality we get that $a=b$. Thus the statement is true for $n+1$. By the principle of mathematical induction this means that it's true for all natural n. $\square$.