(1) Find the normalizers of the following maximal tori in the corresponding Lie groups
  (a) $T^1 = \{e^{it} | t \in \mathbb{R}\} \subset \mathrm{Sp}(1)$.
  (b) $T^1 = \mathrm{SO}(2) \subset \mathrm{SO}(3)$.
  (c) $T^n \subset U(n)$ the set of all diagonal unitary matrices.

(2) (a) Let T be a torus. Prove that every nontrivial real irreducible representation of T is 2-dimensional.
  (b) Let G be a compact connected Lie group with a bi-invariant Riemannian metric. Let $T \subset G$ be a maximal torus and let $\mathfrak{t} = T_eT$ be the Lie algebra of T.
  Look at the $Ad_{G/T}$ action of T on $\mathfrak{t}^\perp \subset \mathfrak{g} = T_eG$. Let V an irreducible summand of this representation and let $v \in V$ be any unit vector.
  Let $t \in T$ be a generator. Prove that the set $\{Ad(t^k)(v) | k \in \mathbb{Z}\}$ is dense in the unit sphere in V.
  (c) Prove that in every compact connected Lie group there exists a dense finitely generated subgroup.
  Hint: Use part b).