(1) Let G be a compact Lie group. Prove that there exists a finite dimensional complex representation of G which is both of real and quaternionic type.

(2) Let G be a compact Lie group such in every dimension up to isomorphism there is at most one irreducible complex representation of G.

Prove that $\chi_V(g) \in \mathbb{R}$ for any complex representation V of G and any $g \in G$.

(3) Verify the formula given in class that $\chi_{V_k}(e(t)) = \frac{\sin(k+1)t}{\sin t}$ (where t is not an integer multiple of π) for the irreducible complex representation V_k of $SU(2)$ and

$$e(t) = \begin{pmatrix} e^{it} & 0 \\ 0 & e^{-it} \end{pmatrix}$$

What is the value of $\chi_{V_k}(e(t))$ when t is an integer multiple of π?