(1) Prove that $U(n)$ is connected.

(2) Let N^3 be the 3-dimensional Heisenberg group

$$N = \{A = \begin{pmatrix} 1 & x & y \\ 0 & 1 & z \\ 0 & 0 & 1 \end{pmatrix} \mid \text{where } x, y, z \in \mathbb{R}\}$$

identified with $\mathbb{R}^3$.

(a) Extend $\frac{\partial}{\partial x}|_0, \frac{\partial}{\partial y}|_0, \frac{\partial}{\partial z}|_0$ to left invariant vector fields X, Y, Z and express them in standard coordinates on $\mathbb{R}^3$.

(b) Compute $[X, Y], [X, Z], [Y, Z]$ and express them both in the standard basis $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$ and in the left invariant basis X, Y, Z.