Random Walks on Groups and Superlinear Divergent Geodesics

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Joint with
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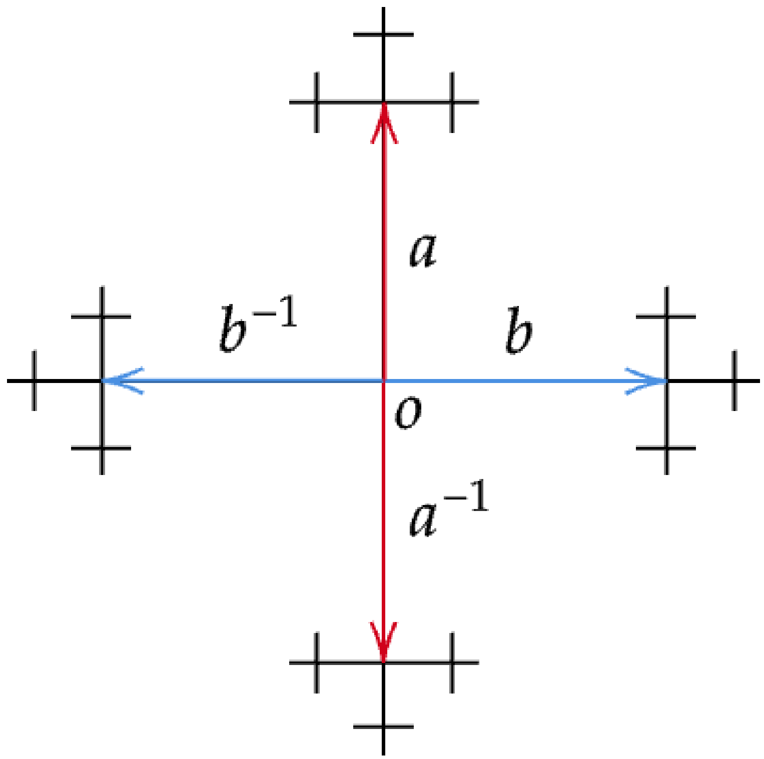
Random Walks on Groups



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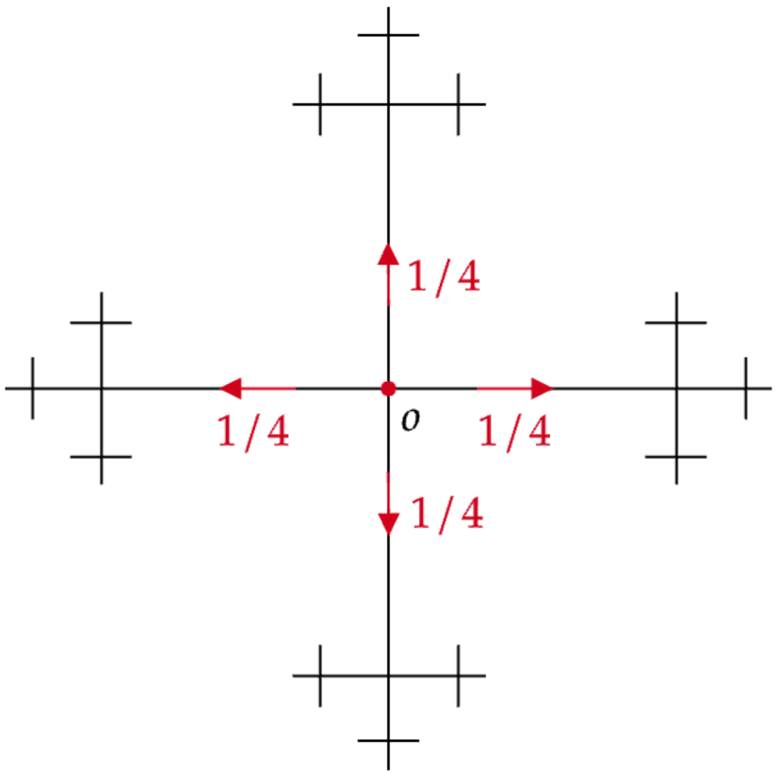

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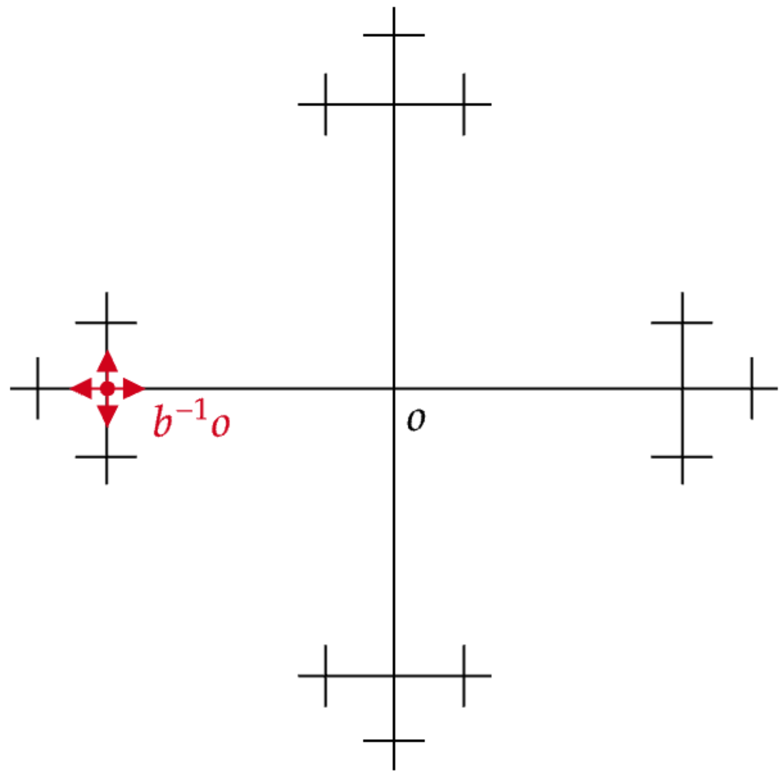

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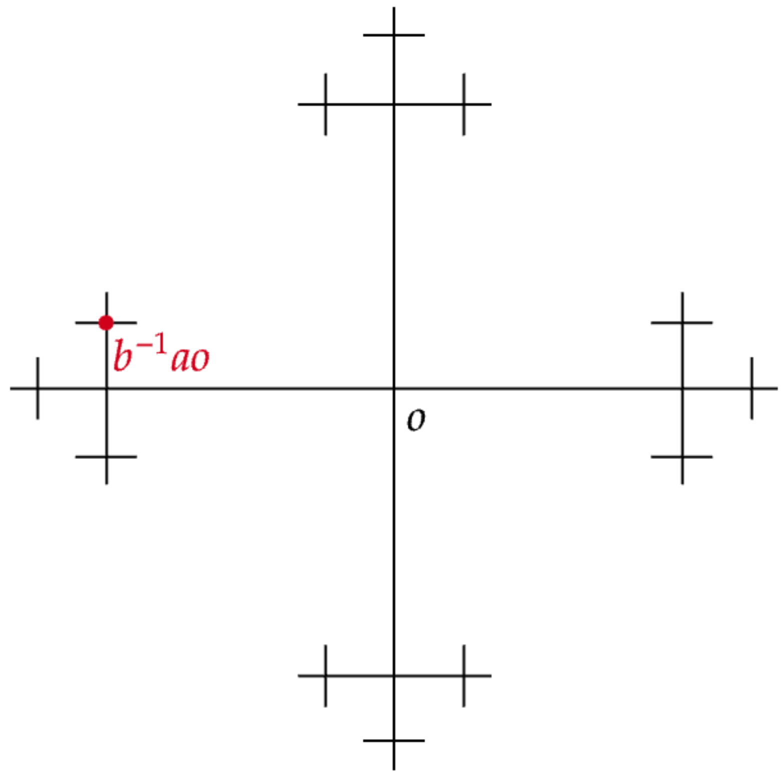

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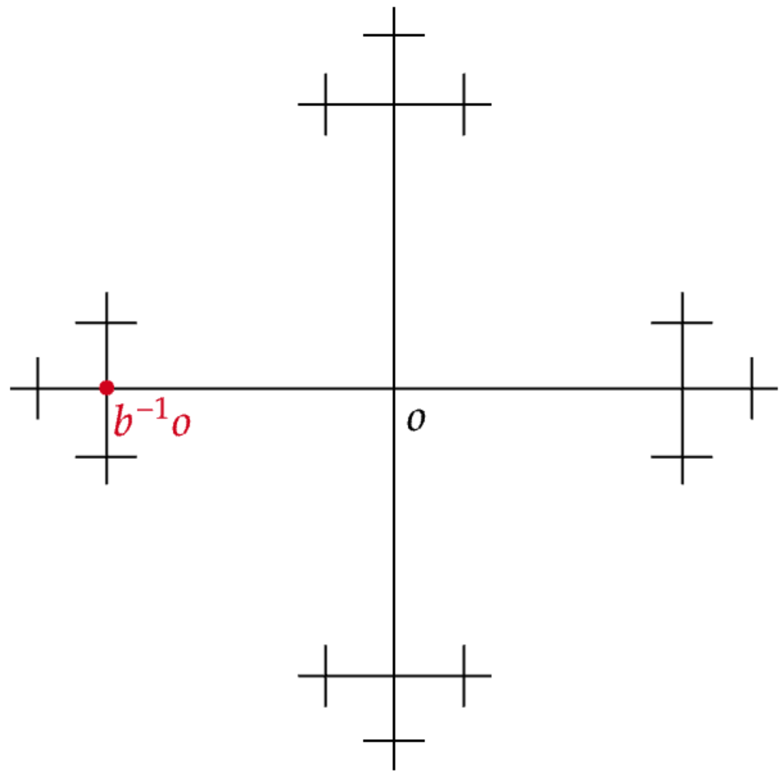

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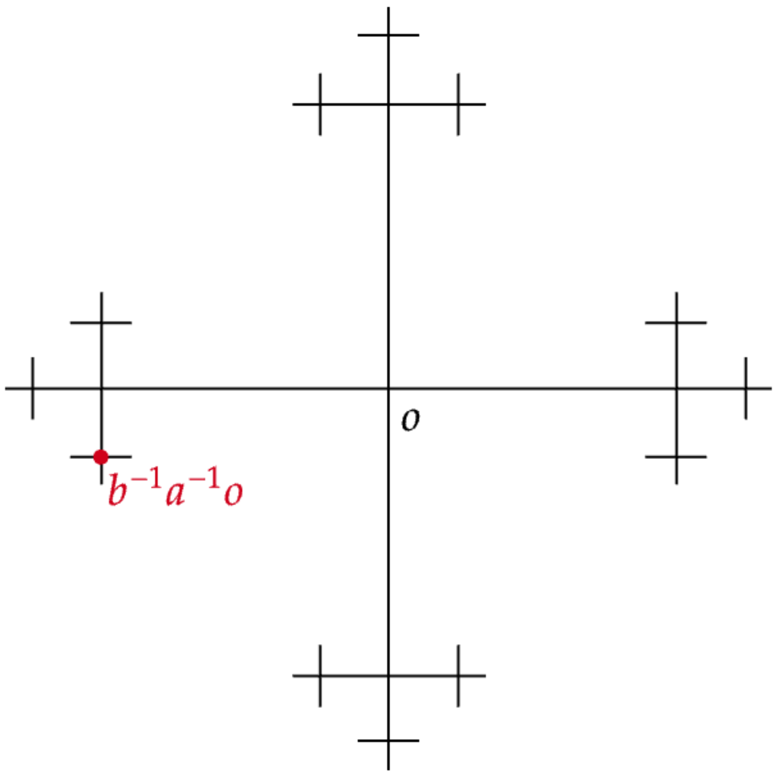

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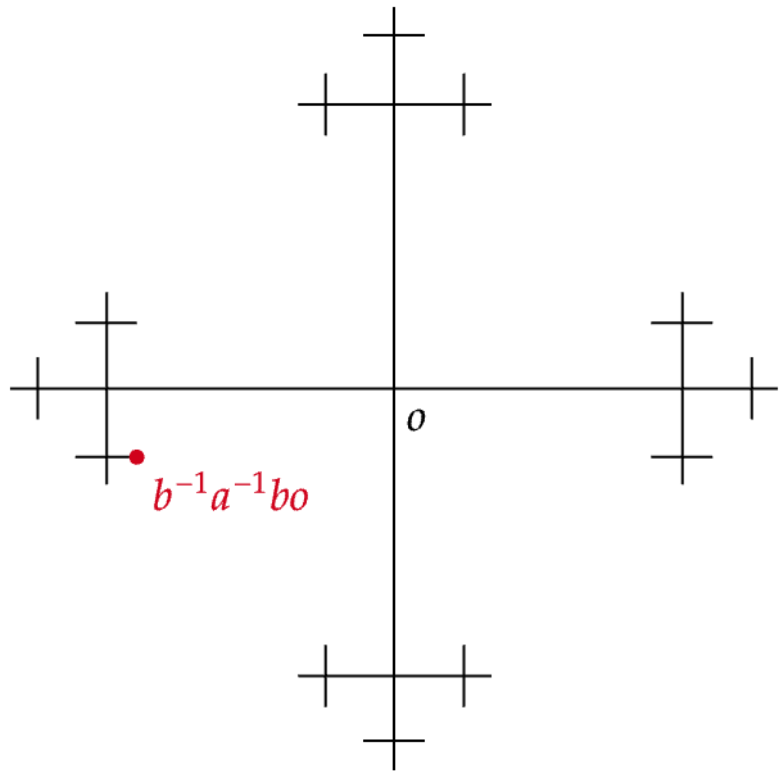
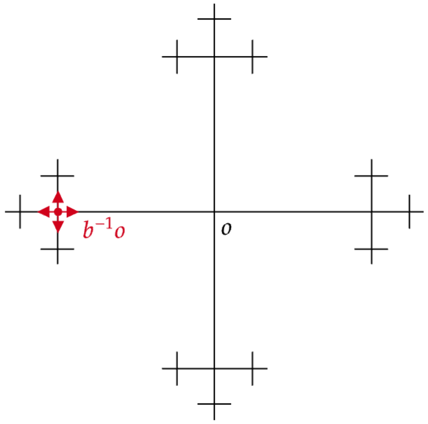


Random Walks on Groups

Observation in this example:

Let d_n denotes the distance at step n. Then

$$d_{n+1} = \begin{cases} d_n + 1 & \text{with probability } 3/4; \\ d_n - 1 & \text{with probability } 1/4. \end{cases}$$



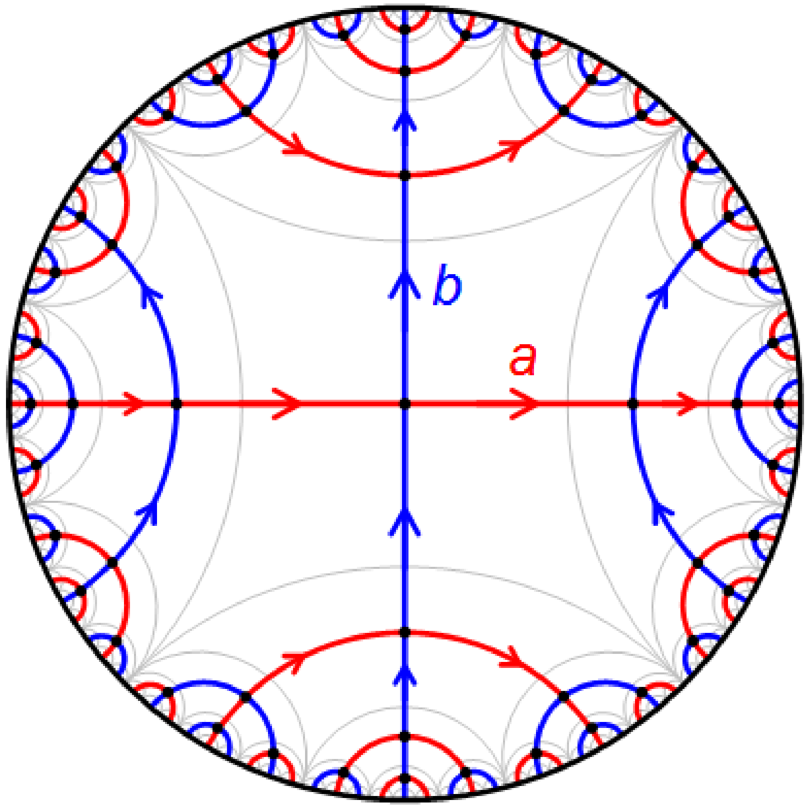
- The distance grows linearly.
- On average, the distance follows a normal distribution (Central Limit Theorem).

Central Limit Theorem

Theorem. *Let G be a finitely generated group [under some conditions]. Let $(Z_n)_{n\geq 1}$ be a simple random walk on G. Then there exist constants λ, σ such that*

$$\frac{d_X(o, Z_n o) - \lambda n}{\sigma\sqrt{n}} \to \mathcal{N}(0,1) \quad \textit{in distribution.}$$

CLT for Hyperbolic Groups



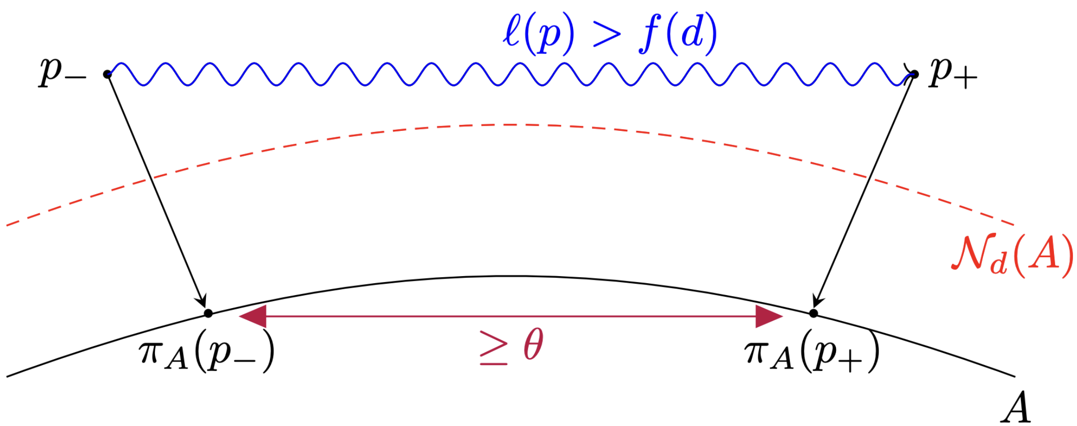
History of CLT

- CLT for free groups **(Sawyer-Steger 1987, Ledrappier 2001)**
- CLT for hyperbolic group **(Bjorkland 2010, Benoist-Quint 2016)**
- CLT for acylindrically hyperboilc groups **(Mathieu-Sisto 2020)**

Superlinear Divergence

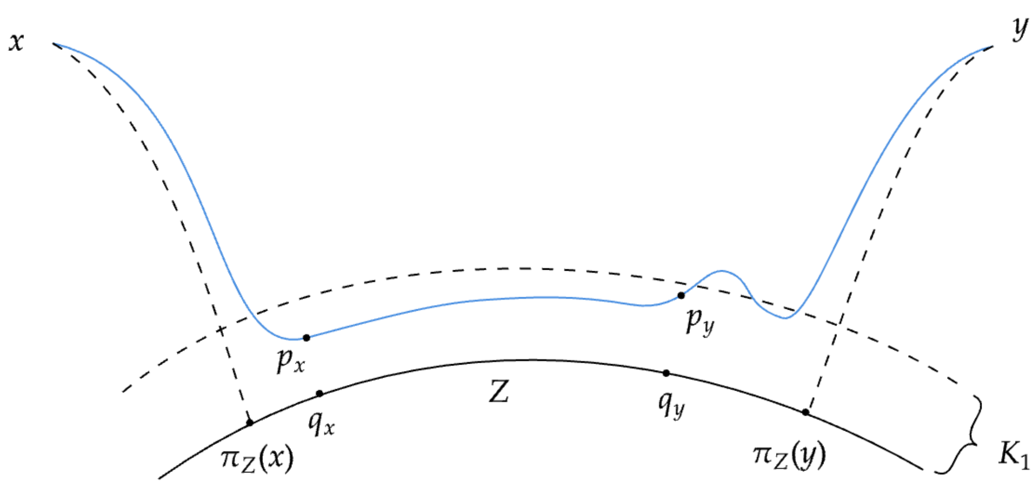
Definition (Goldsborough-Sisto)**.** A subset A is (f,θ)–*divergent* if for any $d>0$ and any path p outside of a d–neighbourhood of A,

$$d(\pi_Z(p_-),\pi_Z(p_+)) > \theta \implies l(p) > f(d).$$

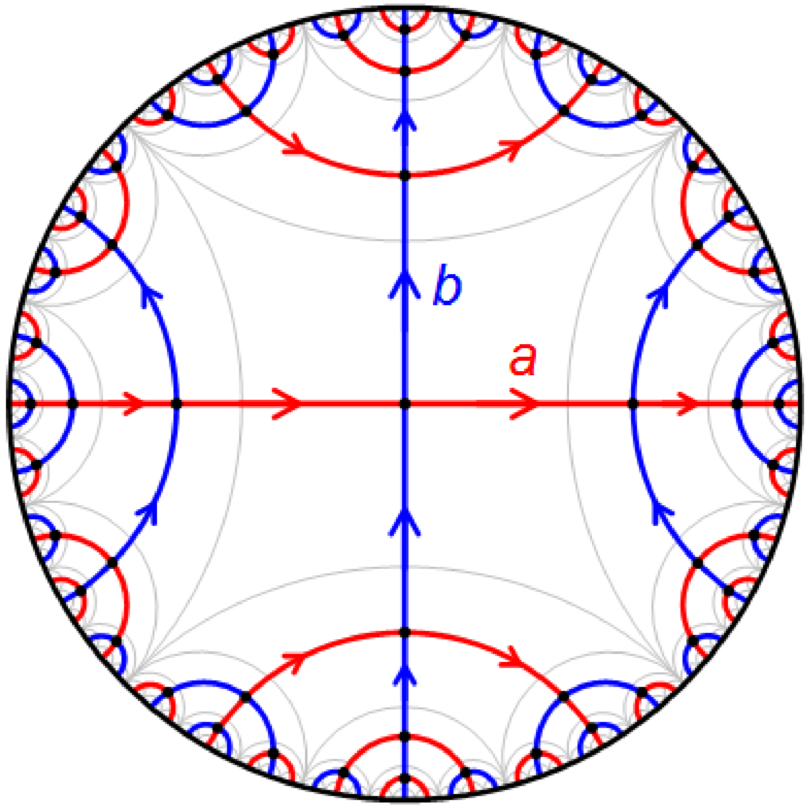


Bottleneck Property

A superliner divergent geodesics "attracts" geodesics.



CLT for Hyperbolic Groups



Central Limit theorem

Theorem (Chawla-Choi-H.-Rafi)**.** *Let G be a finitely generated group with exponential growth, and suppose that G has a superlinear-divergent quasi-geodesic $\gamma : \mathbb{Z} \to G$. Let $(Z_n)_{n \geq 1}$ be a simple random walk on G. Then there exist constants λ, σ such that*

$$\frac{d_X(o, Z_n o) - \lambda n}{\sigma \sqrt{n}} \to \mathcal{N}(0,1) \quad \textit{in distribution.}$$

- The proof uses the pivotal time technique developped by Gouezel.

Thank you!