MATH 1847HS: INTRODUCTION TO HOLOMORPHIC DYNAMICS - WINTER 2021

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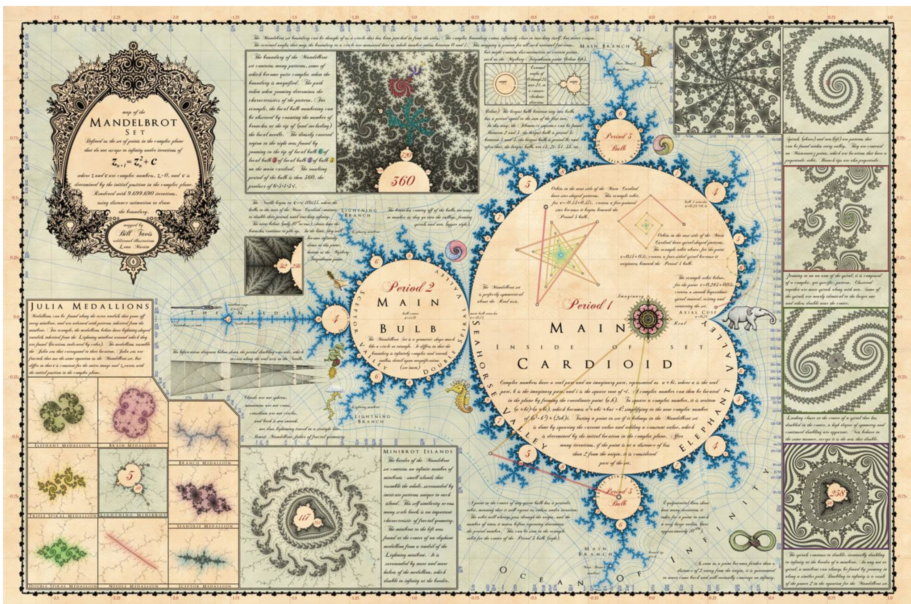

Figure 1. Map of the Mandelbrot set (by Bill Tavis)

The class will focus on holomorphic dynamical systems in one complex variable.

Overview. The theory of iterations of polynomial maps on the plane goes back to the works of Julia and Fatou at the beginning of the 20th century, but it did not fully flourish until the 1980s with the introduction of computer simulations and the works of Douady, Hubbard, Sullivan, Thurston, Milnor, Lyubich, and several others. Very remarkably, even the repeated iteration of a single quadratic polynomial such as $z \mapsto z^2 + c$, where c is a parameter, leads to a very rich fractal geometry, which we will explore. We will investigate the dynamical properties of iterations of polynomials and rational maps on the plane and discuss their classification and their parameter space.

The subject is by now classical, but still full of research questions. We will approach the topic from the beginning, assuming only fundamental knowledge of complex analysis and topology as provided by the core courses. Towards the end, we will also discuss the theory of entropy for real and complex polynomials, a topic I work on and which has been recently revisited by Thurston and still deserves to be fully explored.

Topics.

(1) Riemann surfaces and the Poincare' metric.
(2) The Julia set and the Fatou set.
(3) Local fixed point theory.
(4) Global fixed point theory.
(5) Structure of the Fatou set.
(6) Carathéodory theory and local connectivity.
(7) The Mandelbrot set.
(8) External rays and combinatorics.
(9) Kneading theory.
(10) Entropy of unimodal maps.
(11) Hubbard trees.
(12) Core entropy.

Prerequisites. Graduate complex analysis, real analysis, and topology. We may later encounter some measure theory and some group theory.

Location. Tue 3-5 PM and Thu 4-5 PM. The classes will be held on Zoom. Send me an email if you would like to receive a link to the meeting room.

Instructor. Giulio Tiozzo, `tiozzo@math.utoronto.ca`. Some additional information may be posted on my website `http://www.math.toronto.edu/tiozzo/` especially under *Teaching*.

Office hours. The time for office hours will be decided during the first week of class.

Grading. If you need a grade, let me know during the first class. I will assign you a final project, and you will have to present about it in class and write a short paper. Also, you will be asked to take notes in some of the classes.

Bibliography. The main reference will be

- J. Milnor, *Dynamics in one complex variable*

We will also partially use the following:

- J. Milnor, W. Thurston, *On iterated maps of the interval*
- W. de Melo, S. v. Strien, *One dimensional dynamics*
- C. T. McMullen, *Complex Dynamics and Renormalization*

Later on and for final projects, we will read some of the most recent papers in the field.

Software. I learned a lot about complex dynamics by playing with computers, and I encourage you to do the same. I mostly use the following programs:

- Mandel: `http://www.mndynamics.com/indexp.html`
  You can visualize the Mandelbrot set and explore the related Julia sets, draw external rays and produce polynomials with a given combinatorics.
- FractalStream: `http://www.math.cornell.edu/~noonan/fstream.html`
  You can write small scripts to draw virtually any type of Julia sets you are interested in.

Tentative lecture plan.

(1) (Jan 12) Introduction to the class
(2) (Jan 14) Simply connected Riemann surfaces
(3) (Jan 19) The Universal Covering, Montel's Theorem
(4) (Jan 21) Fatou and Julia: Dynamics on the Riemann Sphere
(5) (Jan 26) Dynamics on Other Riemann Surfaces, Smooth Julia Sets
(6) (Jan 28) Attracting and Repelling Fixed Points
(7) (Feb 2) Parabolic Fixed Points: the Leau-Fatou Flower
(8) (Feb 4) Cremer Points and Siegel Disks
(9) (Feb 9) The Holomorphic Fixed Point Formula; Most Periodic Orbits Repel
(10) (Feb 11) Repelling Cycles are Dense in J
(Feb 16, 18) reading week
(11) (Feb 23) Herman Rings; The Sullivan Classification of Fatou Components
(12) (Feb 25) Sub-hyperbolic and hyperbolic Maps
(13) (Mar 2) Prime Ends; Local Connectivity
(14) (Mar 4) The Filled Julia Set K
(15) (Mar 9) External Rays and Periodic Points
(16) (Mar 11) An overview of the Mandelbrot set
(17) (Mar 16) Kneading theory for unimodal maps
(18) (Mar 18) Entropy of unimodal maps
(19) (Mar 23) Combinatorics of external rays
(20) (Mar 25) Hubbard trees
(21) (Mar 30) The core entropy (I)
(22) (Apr 1) The core entropy (II)